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# New impressive analytical optical soliton solutions to the Schrödinger–Poisson dynamical system against its numerical solutions

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**Abstract**

In our current study, we will derive new diverse enormous impressive analytical optical soliton solutions for the Schrödinger–Poisson dynamical system. The proposed model is applied in gravity field with the corresponding quantum state that produces coupling between different gravity states. Moreover, this model has a significant role in the field of many quantum phenomena. Hereby, we will construct diverse forms of the soliton behaviors that arising from this dynamical system via the solitary wave ansatze method. This technique is one of the ansatze methods that doesn't surrenders to the homogeneous balance and continuously achieves good results. Moreover, we will construct the numerical solutions that are identical for all achieved exact solutions by using two-dimensional differential transform method (TDDTM). The extracted soliton solutions are new compared with that realized before by other authors who used various techniques. The achieved solutions will give new distinct configurations to soliton behaviors arising from this model and show the fact of charges regular distributions on conductors' materials surface.

**Keywords** The Schrödinger–Poisson dynamical system · The solitary wave ansatze method · The two-dimensional differential transform method · The analytical solutions · The numerical solutions

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## 1 Introduction

Schrödinger (Schrödinger 1926) in successive published articles derived the wave equation for time-independent systems that it gave the correct energy eigenvalues for a hydrogen-like atom, solved the quantum harmonic oscillator, rigid rotor, and diatomic molecule problems, proofed consistent to Heisenberg approach and gave the treatment of the Stark effect. Furthermore, he treated problems of time system changes, as in scattering problems. The Schrödinger–Poisson dynamical system (SPDS) is a quantum-model that surrenders to outside quantum operator, gives lose information contained system compared to its environment. From the fact that any quantum system cannot separated from its surroundings, it is important to improve a theoretical formalism treating these interactions to get correct interpretation of quantum systems. The SPDS is essentially the Schrödinger equation as well as the gravitational potential that has application of gravity's role in quantum state in Nakatsuji (2012). Moreover, it can also be seen as the connection of Einstein-Dirac system and Einstein-Klein Gordon in Giulini and Großardt (2012). In addition, it can approximate the coupling between quantum mechanics with gravitation in Penrose (1996). Many models which are derived from Schrödinger equation in various fields of science have been discussed via in Bekir et al. (2020); Bekir et al. 2021; Bekir and Zahran xxxx; Bekir and Zahran 2021a; Shehata et al. 2019; Bekir and Zahran 2021b; Zahran et al. 2021a; Zahran and Bekir 2021; Zahran et al. 2021b; Bekir and Zahran 2020; Bekir and Zahran 2021c; Bekir and Zahran 2021d; Bekir and Zahran 2021e; Bekir and Zahran 2021f; Bekir and Zahran 2021g; Hosseini et al. 2020; Mirzazadeh et al. 2018; Younis et al. 2020; Rehman et al. 2020; Mao et al. 2019; Wang and Wei 2020; Liu et al. 2019; Chen et al. 2021). In related subject, we will use the two-dimensional differential transform method (Zhou 2010; Chen and Ho 1999; Ziyaee and Tari 2015; Ayaz 2003; Patel and Dhodiya 2022) to obtain the numerical solutions corresponding to the achieved analytical solutions. The Schrödinger -Poisson dynamical system (Younis et al. 2021) can be written as;

$$\begin{aligned} -i\frac{\partial \Psi}{\partial t} &= -\Delta\Psi + \Phi(x)\Psi. \\ -\Delta\Phi &= |\Psi|^2. \end{aligned} \tag{1}$$

When this system is surrenders to the definition of $\Delta$ it will be converted to

$$-i\frac{\partial^3\Psi}{\partial x^2\partial t} + \frac{\partial^4\Psi}{\partial x^4} + 2\Phi'\frac{\partial\Psi}{\partial x} - \Phi\frac{\partial^2\Psi}{\partial x^2} + |\Psi|^2\Psi = 0. \tag{2}$$

where $\Psi$ describe the expected shape of the wave pulse, $x, t$ denote to the independent spatial and temporal variables respectively, $\Phi$ which has elliptic behaviors is a solution of Schrödinger–Poisson system, $\Phi(x)$ represent the charge of the wave function itself.

Our article is prepared as follow, in the 1$^{st}$-secession the introduction is present, in the 2$^{nd}$-secession the solitary wave ansatze method (SWAM) (Bekir and Zahran 2020, 2021d; Zahran and Bekir 2022) and its application to detect the bright and dark optical soliton solutions of the suggested model are present. In the 3th-secession the two-dimensional differential transform method and their application to obtain the numerical solutions of the achieved analytical solutions is present. In the 4th-secession we admit the conclusion of our current work (Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10).

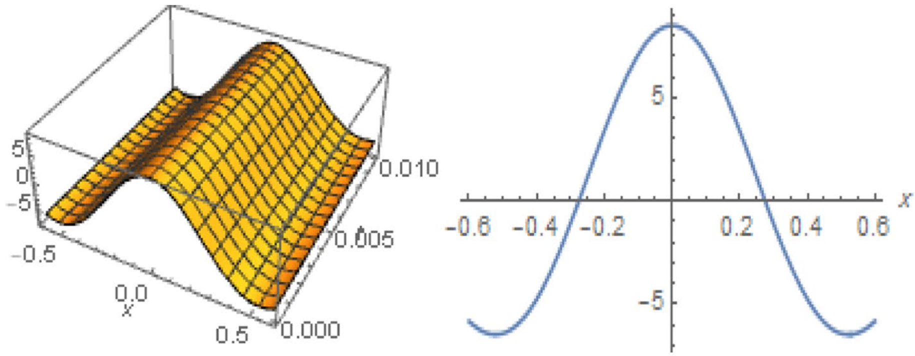


**Fig. 1** The two and three dimensions configuration behavior of the bright solution to the $Re\Psi(x, t)$ in Eq. (31) When $B = 1, \Phi = 33.5, \Omega = 14.5, k = 5.7, w_1 = -22.6, A_1 = 8.5, P = 2$

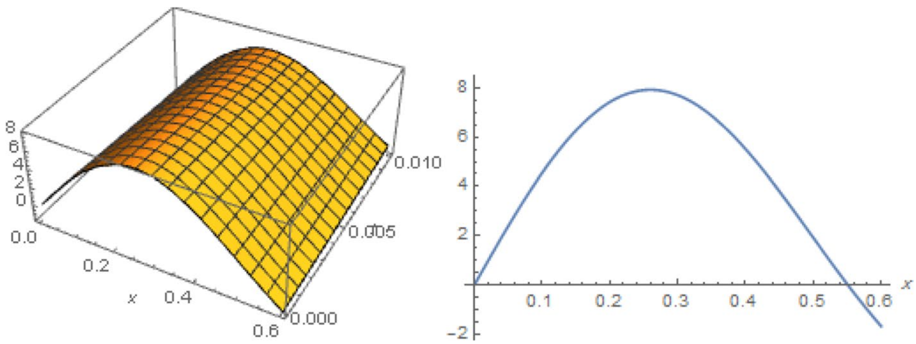


**Fig. 2** The two and three dimensions configuration behavior of the bright solution to the $Im\Psi(x, t)$ in Eq. (32) When $B = 1, \Phi = 33.5, \Omega = 14.5, k = 5.7, w_1 = -22.6, A_1 = 8.5, P = 2$

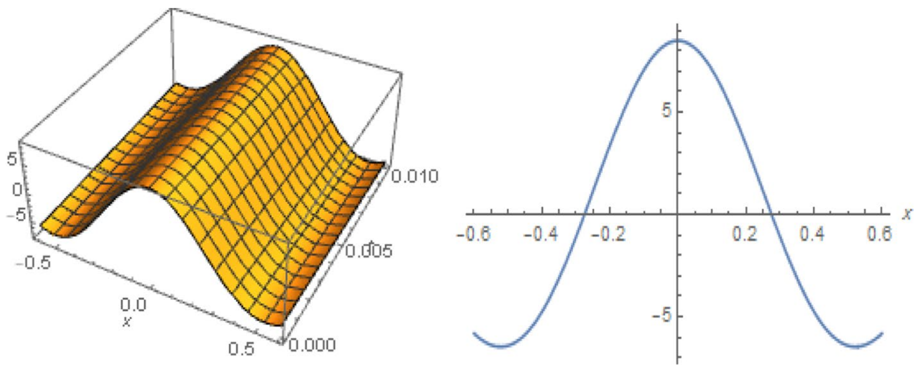


**Fig. 3** The two and three dimensions configuration behavior of the bright solution to the $Re\Psi(x, t)$ in Eq. (35) When $B = 1, \Phi = 33.5, \Omega = 14.5, k = -5.7, w_1 = 22.6, A_1 = 8.5, P = 2$

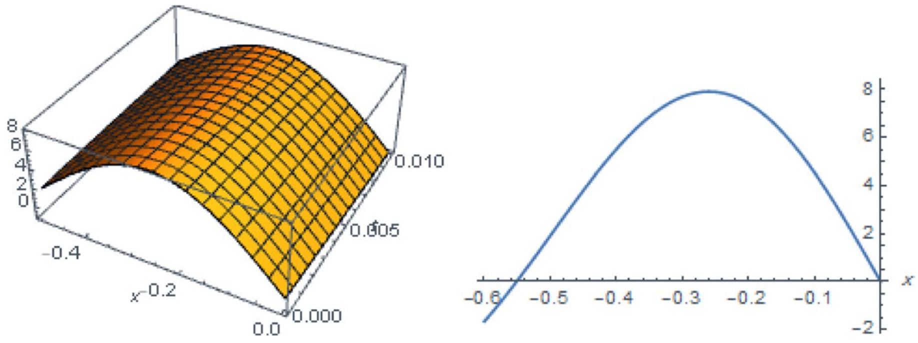


**Fig. 4** The two and three dimensions configuration behavior of the bright solution to the $Im\Psi(x,t)$ in Eq. (36) When $B = 1, \Phi = 33.5, \Omega = 14.5, k = -5.7, w_1 = 22.6, A_1 = 8.5, P = 2$

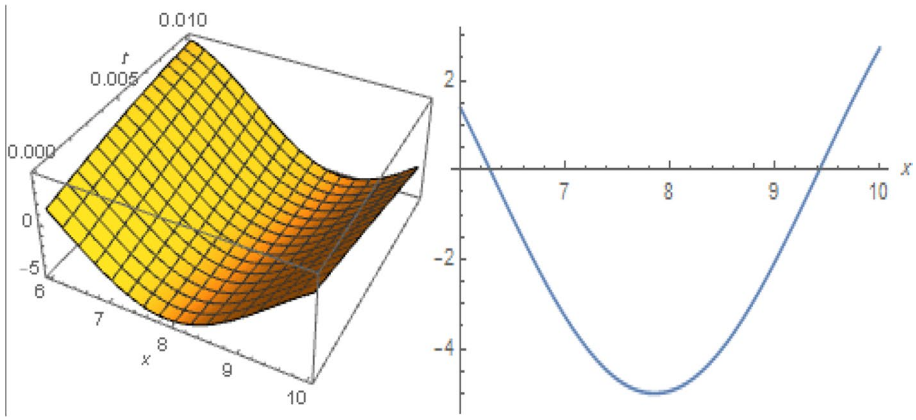


**Fig. 5** The two and three dimension dark solution shape to the $Re\Psi(x,t)$ in Eq. (59) when $\Omega = 91, B = 0.7, A_2 = 5\,i, k = 1, w_2 = -4, P = 2$

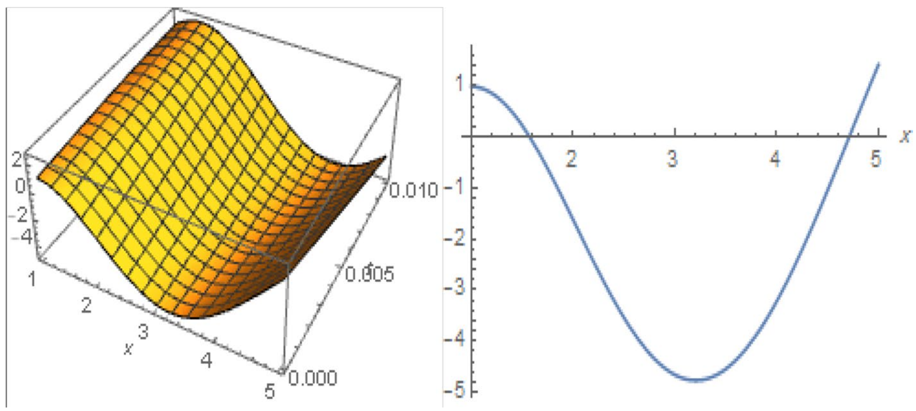


**Fig. 6** The two and three dimension dark solution shape to the $Im\Psi(x,t)$ in Eq. (60) when $\Omega = 91, B = 0.7, A_2 = 5\,i, k = 1, w_2 = -4, P = 2$

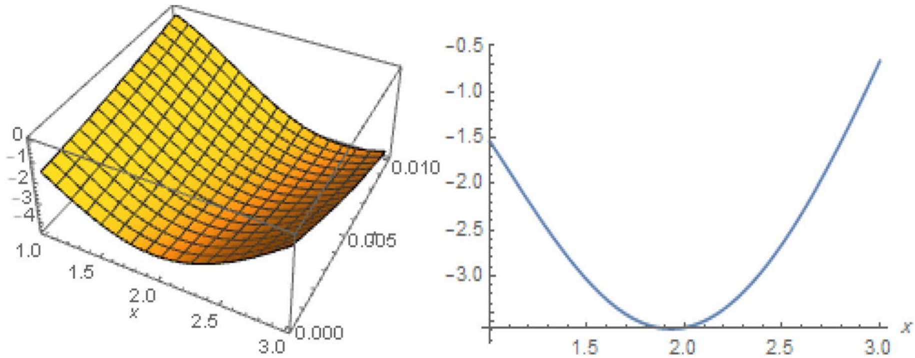


**Fig. 7** The two and three dimension dark solution shape to the $Re\Psi(x,t)$ in Eq. (63) when $\Omega = 91, B = -0.7, A_2 = 5\,i, k = 1, w_2 = -4, P = 2$

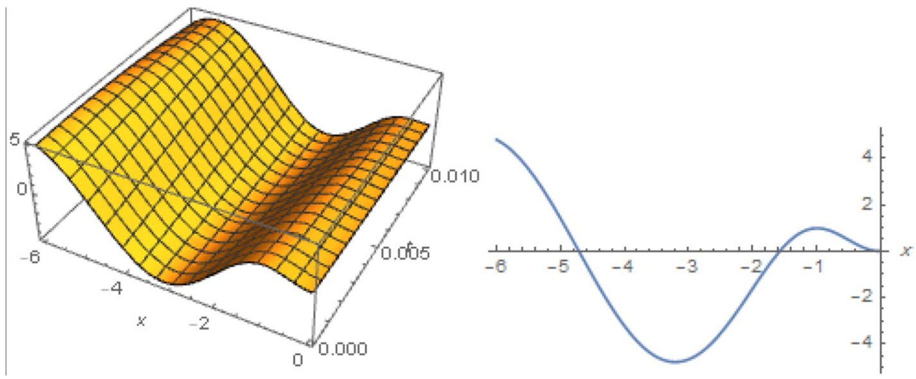


**Fig. 8** The two and three dimension dark solution shape to the $Im\Psi(x,t)$ in Eq. (64) when $\Omega = 91, B = -0.7, A_2 = 5\,i, k = 1, w_2 = -4, P = 2$

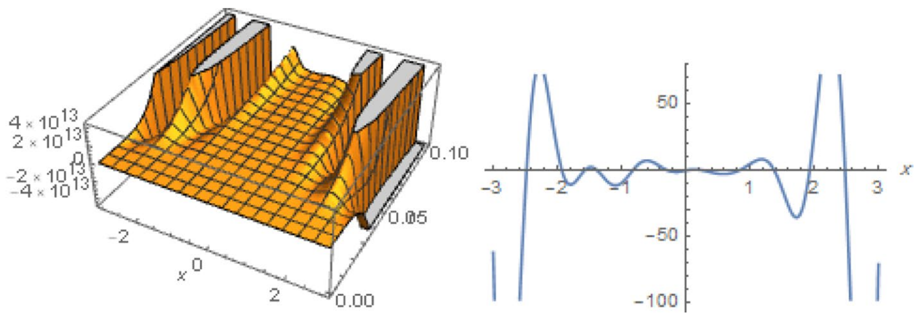


**Fig. 9** The two and three dimension of the numerical solution identical first bright exact solution $Re\Psi(x,t)$ in Eq. (84) when $B = 1, \Phi = 33.5, \Omega = 14.5, k = 5.7, w_1 = -22.6, A_1 = 8.5, P = 2$

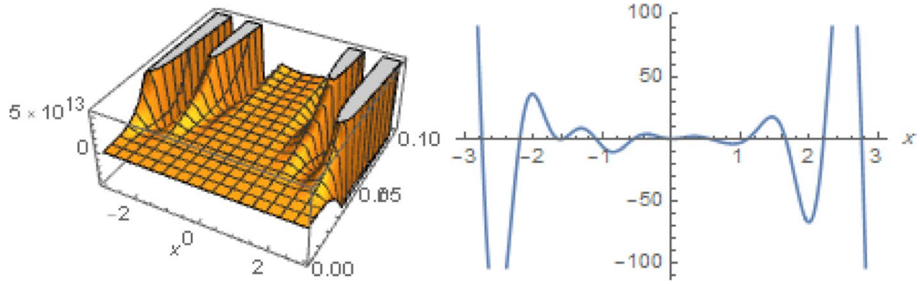


**Fig. 10** The two and three dimension of the numerical solution identical first bright exact solution $Im\Psi(x, t)$ in Eq. (83) when $B = 1, \Phi = 33.5, \Omega = 14.5, k = 5.7, w_1 = -22.6, A_1 = 8.5, P = 2$

## 2 The SWAM schema

The solution perception of the SWAM will be firstly introduced by suppose this transformation

$$\Psi(x, t) = q(x, t)e^{iR(x,t)}, R(x, t) = kx - \Omega t. \tag{3}$$

where $q(x, t)$ represent the portion amplitude,$R(x, t)$ is the portion phase of the soliton, $k, \Omega$ are constants that represent the coefficients of the spatial and temporal $x, t$ of the portion phase respectively, by calculus operations for Eq. (3) we obtain

$$\Psi_t = (q_t - i\Omega q)e^{iR}. \tag{4}$$

$$\Psi_x = (q_x + iqk)e^{iR}. \tag{5}$$

$$\Psi_{xx} = (q_{xx} + 2ikq_x - k^2 q)e^{iR}. \tag{6}$$

$$\Psi_{xxx} = (q_{xxx} + 3ikq_{xx} - ik^3 q - 3k^2 q_x)e^{iR}. \tag{7}$$

$$\Psi_{xxxx} = (q_{xxxx} + 4ikq_{xxx} - 6k^2 q_{xx} + k^4 q - ik^3 q_x)e^{iR}. \tag{8}$$

$$\Psi_{xxt} = \left\{q_{xxt} + 2ikq_{xt} - k^2 q_t - i\Omega q_{xx} + 2\Omega k q_x + iq\Omega k^2\right\}e^{iR}. \tag{9}$$

By inserting the relations (3)–(9) into Eq. (1) leads to

$$\begin{aligned}&-i\left(q_{xxt} + 2ikq_{xt} - k^2 q_t - i\Omega q_{xx} + 2\Omega k q_x + iq\Omega k^2\right)\\&+\left(q_{xxxx} + 4ikq_{xxx} - 6k^2 q_{xx} + k^4 q - ik^3 q_x\right) + 2\Phi'\left(q_x + iqk\right) - \Phi\left(q_{xx} + 2ikq_x - k^2 q\right) + q^3 = 0.\end{aligned} \tag{10}$$

Equation (10) implies these real and imaginary parts;

$$q_{xxxx} + 2kq_{xt} - (\Omega + \Phi + 6k^2)q_{xx} + 2\Phi' q_x + (k^4 - \Phi k^2)q + q^3 = 0. \tag{11}$$

$$4kq_{xxx} - q_{xxt} + k^2 q_t - (2\Omega k + k^3 + 2\Phi k)q_x + 2\Phi' qk = 0. \tag{12}$$

### 2.1 (2-I) The bright solitary wave solution

$$q(x,t) = A_1 \mathrm{sech}^P t_1, \ \ where\, t_1 = B_1(x - w_1 t). \tag{13}$$

where $B_1, w_1$ that appears in the bright solution are constants that represent the coefficients of the spatial and temporal $x, t$ of the portion amplitude respectively and $A_1$ is the coefficient of the amplitude.

$$q_t = A_1 B w_1 P \mathrm{sech}^P t_1 \tanh t_1. \tag{14}$$

$$q_x = -A_1 B P \mathrm{sech}^P t_1 \tanh t_1. \tag{15}$$

$$q_{xx} = -A_1 B^2 P(1+P)\mathrm{sech}^{P+2} t_1 + A_1 B^2 P^2 \mathrm{sech}^P t_1. \tag{16}$$

$$q_{xt} = A_1 w_1 B^2 P(1+P)\mathrm{sech}^{P+2} t_1 - A_1 w_1 B^2 P^2 \mathrm{sech}^P t_1. \tag{17}$$

$$q_{xxx} = A_1 B^3 P(P+1)(P+2)\mathrm{sech}^{P+2} t_1 \tanh\ t_1 - A_1 B^3 P^3 \mathrm{sech}^P t_1 \tanh\ t_1. \tag{18}$$

$$q_{xxt} = -w_1 A_1 B^3 P(P+1)(P+2)\mathrm{sech}^{P+2} t_1 \tanh\ t_1 + w_1 A_1 B^3 P^3 \mathrm{sech}^P t_1 \tanh\ t_1. \tag{19}$$

$$q_{xxxx} = A_1 B^4 P(P+1)(P+2)^2 \mathrm{sech}^{P+2} t_1 - A_1 B^4 P(P+1)^2(P+2)\mathrm{sech}^{P+4} t_1 - A_1 B^4 P^4 \mathrm{sech}^P t_1. \tag{20}$$

By introducing the relations (13)–(20) into the real and imaginary parts Eqs. (11), (12) we get

$$\begin{aligned} &\left(A_1 B^4 P(P+1)(P+2)^2 \mathrm{sech}^{P+2} t_1 - A_1 B^4 P(P+1)^2(P+2)\mathrm{sech}^{P+4} t_1 - A_1 B^4 P^4 \mathrm{sech}^P t_1\right) \\ &+ 2k\left(A_1 w_1 B^2 P(1+P)\mathrm{sech}^{P+2} t_1 - A_1 w_1 B^2 P^2 \mathrm{sech}^P t_1\right) \\ &- (\Omega + \Phi + 6k^2)\left(-A_1 B^2 P(1+P)\mathrm{sech}^{P+2} t_1 + A_1 B^2 P^2 \mathrm{sech}^P t_1\right) \\ &+ 2\Phi'\left(-A_1 B P \mathrm{sech}^P t_1 \tanh t_1\right) + (k^4 - \Phi k^2)\left(A_1 \mathrm{sech}^P t_1\right) + A_1^3 \mathrm{sech}^{3P} t_1 = 0. \end{aligned} \tag{21}$$

$$\begin{aligned} &4k\left(A_1 B^3 P(P+1)(P+2)\mathrm{sech}^{P+2} t_1 \tanh\ t_1 - A_1 B^3 P^3 \mathrm{sech}^P t_1 \tanh\ t_1\right) \\ &-\left(-w_1 A_1 B^3 P(P+1)(P+2)\mathrm{sech}^{P+2} t_1 \tanh\ t_1 + w_1 A_1 B^3 P^3 \mathrm{sech}^P t_1 \tanh\ t_1\right) \\ &+k^2\left(A_1 B w_1 P \mathrm{sech}^P t_1 \tanh t_1\right) - (2\Omega k + k^3 + 2\Phi k)\left(-A_1 B P \mathrm{sech}^P t_1 \tanh t_1\right) \\ &+2\Phi' k\left(A_1 \mathrm{sech}^P t_1\right) = 0. \end{aligned} \tag{22}$$

The equivalence between the higher orders of $\mathrm{sech}^i$ in Eq. (21) implies $P = 2$, by substituting about this value into Eq. (21) and equating the coefficients of various powers of $\mathrm{sech}^i$ to zero we obtain the following relations:

$$\begin{aligned} &-4B\Phi' = 0. \\ &A_1^2 - 72B^4 = 0. \\ &96B^2 + 12kw_1 + 6(\Omega + \Phi + 6k^2) = 0. \\ &k^4 - \Phi k^2 - 4B^2(\Omega + \Phi + 6k^2) - 8w_1 B^2 k - 16B^4 = 0. \end{aligned} \tag{23}$$

By put $P = 2$ in the imaginary part Eq. (22) we get the following relations

$$\begin{aligned} &2(k^2 - 4B^2)w_1 + 2(2\Omega k + k^3 + 2\Phi k) - 32kB^2 = 0. \\ &2\Phi' k = 0. \\ &4k + w_1 = 0. \end{aligned} \tag{24}$$

It is clear that the two systems given by Eqs. (23) and (24) imply that $\Phi' = 0$ which leads to $\Phi = C$ which physically denotes to the regular distribution of the charge that must occur in the conducting materials. Moreover, when the two systems in Eqs. (23) and (24) are merged with each other and solving the resultant system the following results are detected. While, from the first part we can extract

$$(1)\Omega = \frac{29B^2}{2}, \Phi = \frac{67B^2}{2}, k = 4\sqrt{2}B, w_1 = -16\sqrt{2}B, A_1 = 6\sqrt{2}B^2. \tag{25}$$

$$(2)\Omega = \frac{29B^2}{2}, \Phi = \frac{67B^2}{2}, k = 4\sqrt{2}B, w_1 = -16\sqrt{2}B, A_1 = -6\sqrt{2}B^2. \tag{26}$$

$$(3)\Omega = \frac{29B^2}{2}, \Phi = \frac{67B^2}{2}, k = -4\sqrt{2}B, w_1 = 16\sqrt{2}B, A_1 = 6\sqrt{2}B^2. \tag{27}$$

$$(4)\Omega = \frac{29B^2}{2}, \Phi = \frac{67B^2}{2}, k = -4\sqrt{2}B, w_1 = 16\sqrt{2}B, A_1 = -6\sqrt{2}B^2. \tag{28}$$

We will construct the solutions that are identical only for the first and the third results after putting $B = 1$.

**(1) For the first result which is**

$$\Omega = \frac{29B^2}{2}, \Phi = \frac{67B^2}{2}, k = 4\sqrt{2}B, w_1 = -16\sqrt{2}B, A_1 = 6\sqrt{2}B^2.$$

This result can be simplified to be

$$B = 1, \Phi = 33.5, \Omega = 14.5, k = 5.7, w_1 = -22.6, A_1 = 8.5. \tag{29}$$

Thus the solution is

$$\Psi(\zeta) = A_1 \mathrm{sech}^2[x - w_1 t]e^{i(kx - \Omega t)}$$

$$\Psi(x, t) = 8.5\mathrm{sech}^2[x + 22.6t]e^{i(5.7x - 14.5t)}. \tag{30}$$

$$\mathrm{Re}\Psi(x, t) = 8.5\left(\mathrm{sech}^2[x + 22.6t]\right) \times \cos(5.7x - 14.5t) \tag{31}$$

$$\mathrm{Im}\Psi(x, t) = 8.5\left(\mathrm{sech}^2[x + 22.6t]\right) \times \sin(5.7x - 14.5t) \tag{32}$$

(2) For the third result which is:

$$\Omega = \frac{29B^2}{2}, \Phi = \frac{67B^2}{2}, k = -4\sqrt{2}B, w_1 = 16\sqrt{2}B, A_1 = 6\sqrt{2}B^2.$$

This result can be simplified to be:

$$B = 1, \Phi = 33.5, \Omega = 14.5, k = -5.7, w_1 = 22.6, A_1 = 8.5 \tag{33}$$

Thus the solution is:

$$\Psi(\zeta) = A_1 \mathrm{sech}^2[x - w_1 t] e^{i(kx - \Omega t)}.$$

$$\Psi(x,t) = 8.5\mathrm{sech}^2[x - 22.6t] e^{i(-5.7x - 14.5t)}.$$

$$\mathrm{Re}\Psi(x,t) = 8.5\left(\mathrm{sech}^2[x - 22.6t]\right) \times \cos(5.7x + 14.5t) \tag{34}$$

$$\mathrm{Im}\Psi(x,t) = -8.5\left(\mathrm{sech}^2[x - 22.6t]\right) \times \sin(5.7x + 14.5t) \tag{35}$$

By the same method we can plot solutions that are corresponding to the other two results.

## 2.2 The dark solitary wave solution

$$q(x,t) = A_2 \tanh^P t_2, \ \ where\, t_2 = B(x - w_2 t). \tag{36}$$

where $B_2, w_2$ that appears in the dark solution are constants that represent the coefficients of the spatial and temporal $x, t$ of the portion amplitude respectively and $A_2$ is the coefficient of the amplitude

$$q_t = -A_2 B P w_2 [\tanh^{P-1} t_2 - \tanh^{P+1} t_2]. \tag{37}$$

$$q_x = A_2 B P [\tanh^{P-1} t_2 - \tanh^{P+1} t_2]. \tag{38}$$

$$q_{xx} = A_2 P(P-1) B^2 \tanh^{P-2} t_2 - 2A_2 P^2 B^2 \tanh^P t_2 + A_2 P(P+1) B^2 \tanh^{P+2} t_2. \tag{39}$$

$$q_{xt} = -w_2 A_2 P(P-1) B^2 \tanh^{P-2} t_2 + 2w_2 A_2 P^2 B^2 \tanh^P t_2 - w_2 A_2 P(P+1) B^2 \tanh^{P+2} t_2. \tag{40}$$

$$\begin{aligned} q_{xxx} = {} & A_2 B^3 P(P-1)(P-2) \tanh^{P-3} t_2 - [A_2 B^3 P(P-1)(P-2) + 2A_2 P^3 B^3] \tanh^{P-1} t_2 + \\ & [A_2 B^3 P(P+1)(P+2) + 2A_2 P^3 B^3] \tanh^{P+1} t_2 - A_2 B^3 P(P+1)(P+2) \tanh^{P+3} t_2. \end{aligned} \tag{41}$$

$$\begin{aligned} q_{xxxx} = {} & A_2 B^4 P(P+1)(P+2)(P+3) \tanh^{P+4} t_2 + A_2 B^4 P(P-1)(P-2)(P-3) \tanh^{P-4} t_2 \\ & + 2A_2 B^4 P(P+1)^2 (P-2) \tanh^{P+2} t_2 - A_2 B^4 P(P-1)(3P^2 + 8P + 8) \tanh^{P-2} t_2 \\ & - A_2 B^4 P(P^4 + 4P^3 + 17P^2 + 2P + 4) \tanh^P t_2. \end{aligned} \tag{42}$$

$$q_{xxt} = -w_2A_2B^3P(P-1)(P-2)\tanh^{P-3}t_2 - [-w_2A_2B^3P(P-1)(P-2) + 2A_2P^3B^3]\tanh^{P-1}t_2 + [-w_2A_2B^3P(P+1)(P+2) + 2A_2P^3B^3]\tanh^{P+1}t_2 + w_2A_2B^3P(P+1)(P+2)\tanh^{P+3}t_2. \tag{43}$$

By introducing the relations (37–44) into the real and imaginary parts Eqs. (11), (12) we get

$$\begin{aligned}&\left\{\begin{array}{l}A_2B^4P(P+1)(P+2)(P+3)\tanh^{P+4}t_2 + A_2B^4P(P-1)(P-2)(P-3)\tanh^{P-4}t_2\\ +2A_2B^4P(P+1)^2(P-2)\tanh^{P+2}t_2 - A_2B^4P(P-1)(3P^2+8P+8)\tanh^{P-2}t_2\\ -A_2B^4P(P^4+4P^3+17P^2+2P+4)\tanh^{P}t_2\end{array}\right\}\\ &+2k\left(-w_2A_2P(P-1)B^2\tanh^{P-2}t_2 + 2w_2A_2P^2B^2\tanh^{P}t_2 - w_2A_2P(P+1)B^2\tanh^{P+2}t_2\right)\\ &-(\Omega+\Phi+6k^2)\left(A_2P(P-1)B^2\tanh^{P-2}t_2 - 2A_2P^2B^2\tanh^{P}t_2 + A_2P(P+1)B^2\tanh^{P+2}t_2\right)\\ &+2\Phi'\left(A_2BP[\tanh^{P-1}t_2 - \tanh^{P+1}t_2]\right) + (k^4-\Phi k^2)\left(A_2\tanh^{P}t_2\right) + A_2^3\tanh^{3P}t_2 = 0.\end{aligned} \tag{44}$$

$$\begin{aligned}&4k\left\{\begin{array}{l}A_2B^3P(P-1)(P-2)\tanh^{P-3}t_2 - [A_2B^3P(P-1)(P-2) + 2A_2P^3B^3]\tanh^{P-1}t_2\\ +[A_2B^3P(P+1)(P+2) + 2A_2P^3B^3]\tanh^{P+1}t_2 - A_2B^3P(P+1)(P+2)\tanh^{P+3}t_2\end{array}\right\}\\ &-\left\{\begin{array}{l}-w_2A_2B^3P(P-1)(P-2)\tanh^{P-3}t_2 - [-w_2A_2B^3P(P-1)(P-2) + 2A_2P^3B^3]\tanh^{P-1}t_2\\ +[-w_2A_2B^3P(P+1)(P+2) + 2A_2P^3B^3]\tanh^{P+1}t_2 + w_2A_2B^3P(P+1)(P+2)\tanh^{P+3}t_2\end{array}\right\}\\ &+k^2\left(-A_2BPw_2[\tanh^{P-1}t_2 - \tanh^{P+1}t_2]\right) - (2\Omega k + k^3 + 2\Phi k)\left(A_2BP[\tanh^{P-1}t_2 - \tanh^{P+1}t_2]\right)\\ &+2\Phi' k\left(A_2\tanh^{P}t_2\right) = 0.\end{aligned} \tag{45}$$

By equating the coefficients of various powers of $\mathrm{sech}^i$ in Eq. (45) to zero, it implies $P = 2$, by substituting this value into Eq. (45) again, this leads to the following relations:

$$\begin{aligned}&-4kw_2B^2 - 72B^4 - 2B^2(\Omega+\Phi+6k^2) = 0.\\ &120B^4 + A_2^2 = 0.\\ &-(\Omega+\Phi+6k^2) + 2kw_2 = 0.\\ &k^4 - \Phi k^2 + 8(\Omega+\Phi+6k^2)B^2 + 16kw_2B^2 - 248B^4 = 0.\\ &4\Phi' B = 0.\end{aligned} \tag{46}$$

By put $P = 2$ in the imaginary part Eq. (46) we get

$$2\Phi' k = 0, 4k + w_2 = 0. \tag{47}$$

The resultant relations Eqs. (47) and (48) imply $\Phi' = 0$ and by substituting from Eq. (48) into Eq. (47) about $4k + w_2 = 0, \Phi' = 0$ we obtain the following system of equations

$$16k^2B^2 - 72B^4 - 2B^2(\Omega+\Phi+6k^2) = 0. \tag{48}$$

$$120B^4 + A_2^2 = 0. \tag{49}$$

$$-(\Omega+\Phi+6k^2) - 2k^2 = 0. \tag{50}$$

$$k^4 - \Phi k^2 + 8(\Omega+\Phi+6k^2)B^2 - 64k^2B^2 - 248B^4 = 0. \tag{51}$$

When this system is solved by any computer program the following results will be detected:

$$(1)\Phi = \frac{-8495k^2}{81}, \Omega = \frac{7361k^2}{81}, B = \frac{2k}{3}, A_2 = \frac{8}{3}\sqrt{\frac{10}{3}}\, ik^2, w_2 = -4k. \tag{52}$$

$$(2)\Phi = \frac{-8495k^2}{81}, \Omega = \frac{7361k^2}{81}, B = \frac{2k}{3}, A_2 = -\frac{8}{3}\sqrt{\frac{10}{3}}\, ik^2, w_2 = -4k. \tag{53}$$

$$(3)\Phi = \frac{-8495k^2}{81}, \Omega = \frac{7361k^2}{81}, B = -\frac{2k}{3}, A_2 = \frac{8}{3}\sqrt{\frac{10}{3}}\, ik^2, w_2 = -4k. \tag{54}$$

$$(4)\Phi = \frac{-8495k^2}{81}, \Omega = \frac{7361k^2}{81}, B = -\frac{2k}{3}, A_2 = -\frac{8}{3}\sqrt{\frac{10}{3}}\, ik^2, w_2 = -4k. \tag{55}$$

For the above achieved results, we will extract the solutions that are identical for the first and the third one.

(1) **For the first result which is:**

$$\Phi = \frac{-8495k^2}{81}, \Omega = \frac{7361k^2}{81}, B = \frac{2k}{3}, A_2 = \frac{8}{3}\sqrt{\frac{10}{3}}\, ik^2, w_2 = -4k.$$

This can be simplified to be:

$$\Omega = 91, B = 0.7, A_2 = 5\,i, k = 1, w_2 = -4. \tag{56}$$

The dark solution is:

$$\Psi(x,t) = A_2 \tanh^{R_2}[B(x - w_2 t)]e^{i(kx-\Omega t)}.$$

$$\Psi(x,t) = 5i \tanh^2[0.7(x + 4t)]e^{i(x-91t)}. \tag{57}$$

$$\mathrm{Re}\Psi(x,t) = -5 \tanh^2[0.7(x + 4t)] \sin(x - 91t). \tag{58}$$

$$\mathrm{Im}\Psi(x,t) = 5 \tanh^2[0.7(x + 4t)] \cos(x - 91t) \tag{59}$$

(2) **For the third result which is:**

$$\Phi = \frac{-8495k^2}{81}, \Omega = \frac{7361k^2}{81}, B = -\frac{2k}{3}, A_2 = \frac{8}{3}\sqrt{\frac{10}{3}}\, ik^2, w_2 = -4k.$$

This can be simplified to be:

$$\Omega = 91, B = -0.7, A_2 = 5\,i, k = 1, w_2 = -4. \tag{60}$$

The dark solution is:

$$\Psi(x,t) = A_2 \tanh^{R_2}[B(x - w_2 t)]e^{i(kx-\Omega t)}.$$

$$\Psi(x,t) = 5i\tanh^2[-0.7(x+4t)]e^{i(x-91t)}. \tag{61}$$

$$\mathrm{Re}\Psi(x,t) = -5\tanh^2[-0.7(x+4t)]\sin(x-91t). \tag{62}$$

$$\mathrm{Im}\Psi(x,t) = 5\tanh^2[-0.7(x+4t)]\cos(x-91t). \tag{63}$$

The other two solutions can be implemented by the same technique.

## 3 Numerical treatment for the Schrödinger–Poisson dynamical system using two dimensional differential transform method (TDDTM)

The idea of differential transform was first introduced by Zhou (Patel and Dhodiya 2022), who used it to solve linear and nonlinear initial value problems in electric circuit analysis. Chen and Ho (Younis et al. 2021) extended this method for partial differential equations (PDEs) and got on a closed form series solution for linear and nonlinear initial value problems. The method applied to PDEs, is called two-dimensional differential transform method (TDDTM) (Bekir and Zahran 2020, 2021d; Zahran and Bekir 2022), this method is different from the high-order Taylor series method, which consists of computing the coefficients of the Taylor series of the solution using the initial data and the partial differential equation. But the Taylor series method needs more computational work for large orders. It is stated that ‘‘the differential transform is an iterative procedure for obtaining Taylor series solutions of differential equations’’. This method decreases the size of computational domain and suitable to many problems easily.

### 3.1 The two-dimensional differential transform method (TDDTM)

TDDTM is defined as

$$q(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} Q(k,h)x^k t^h \tag{64}$$

where

$$Q(k,h) = \frac{1}{k!h!}\left[\frac{\partial^{k+h}q(x,t)}{\partial x^k \partial t^h}\right]_{(0,0)} \tag{65}$$

where $q(x,t)$ is the original function and $Q(k,h)$ is the transformed function. Using Eq. (66) in Eq. (65) we obtain

$$q(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!h!}\left[\frac{\partial^{k+h}q(x,t)}{\partial x^k \partial t^h}\right]_{(0,0)} x^k t^h \tag{66}$$

The following relations (rules) are important for solution, (Bekir and Zahran 2020).

(1) If $c(x,t) = \frac{\partial u(x,t)}{\partial x}$ then $C(k,h) = (k+1)U(k+1,h)$
(2) If $c(x,t) = \frac{\partial u(x,t)}{\partial t}$ then $C(k,h) = (h+1)U(k,h+1)$
(3) If $c(x,t) = \frac{\partial^r u(x,t)}{\partial x^r}$ then

$$C(k,h) = (k+1)(k+2)\dots(k+r)U(k+r,h)(68) \tag{67}$$

(4) If $c(x,t) = \frac{\partial^s u(x,t)}{\partial t^s}$ then

$$C(k,h) = (h+1)(h+2)\dots(h+s)U(k,h+s)(69) \tag{68}$$

(5) If $c(x,t) = \frac{\partial^{r+s} u(x,t)}{\partial x^r \partial t^s}$ then

$$C(k,h) = (k+1)(k+2)\dots(k+r)(h+1)(h+2)\dots(h+s)U(k+r,h+s)(70) \tag{69}$$

(6) If $c(x,t) = u(x,t) * v(x,t)$then

$$C(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)V(k-r,s) \tag{70}$$

(7) If $c(x,t) = u(x,t) * v(x,t) * w(x,t)$ then

$$C(k,h) = \sum_{r=0}^{k}\sum_{\tau=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(r,h-s-p)V(\tau,s)W(k-r-\tau,p) \tag{71}$$

## 4 TDDTM solution for the real part of the Schrödinger–Poisson dynamical system

Consider the real part of the Schrödinger–Poisson dynamical system Eq. (11)

$$q_{xxxx} + 2kq_{xt} - \left(\Omega + \Phi + 6k^2\right)q_{xx} + 2\Phi' q_x + \left(k^4 - \Phi k^2\right)q + q^3 = 0$$

Taking the same values of the parameters given before $k = 5.7, \Omega = 14.5, \Phi = 33.5, \Phi' = 0k = 5.7, \Omega = 14.5, \Phi = 33.5, \Phi' = 0$ then the previous equation becomes,

$$q_{xxxx} + 11.4q_{xt} - 242.94q_{xx} - 32.82q + q^3 = 0 \tag{72}$$

By using the relations given in rule (1) we have

$$q_{xxxx} \to (k+1)(k+2)(k+3)(k+4)Q(k+4,h)\&q_{xt} \to (k+1)(h+1)Q(k+1,h+1)$$

$$\&q_{xx} \to (k+1)(k+2)Q(k+2,h)\&q(x,t) \to Q(k,h) \tag{73}$$

$$\&q^3 = q(x,t)q(x,t)q(x,t)$$

$$\to F(k,h) = \sum_{r=0}^{k}\sum_{\tau=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(r,h-s-p)V(\tau,s)W(k-r-\tau,p) \tag{74}$$

Substituting from Eq. (74) and Eq. (75) in Eq. (73) we have

$$
\begin{aligned}
&(k+1)(k+2)(k+3)(k+4)Q(k+4,h)\\
&+11.4(k+1)(h+1)Q(k+1,h+1)\\
&-242.94(k+1)(k+2)Q(k+2,h)\\
&-32.82Q(k,h)+F(k,h)=0
\end{aligned}
$$

And the recurrence relation takes the form

$$Q(k+4,h)=\frac{-11.4(k+1)(h+1)Q(k+1,h+1)+242.94(k+1)(k+2)Q(k+2,h)+32.82Q(k,h)-F(k,h)}{(k+1)(k+2)(k+3)(k+4)} \tag{75}$$

Taking the exact first solution reduced from the bright solitary wave,

$$q(x,t)=8.5\left(\sec h^2[x+22.6t]\right) \tag{76}$$

Using Eqs. (76) and (65) to find the initial conditions

$$Q(0,0)=\frac{1}{0!0!}\left[\frac{\partial^0 q(x,t)}{\partial x^0\partial t^0}\right]_{(0,0)}=q(0,0)=8.5 \tag{77}$$

$$Q(1,0)=\frac{1}{1!0!}\left[\frac{\partial q(x,t)}{\partial x}\right]_{(0,0)}=-17\left(\sec h^2[x+22.6t]\right)\tanh[x+22.6t]=0 \tag{78}$$

$$Q(0,1)=\frac{1}{0!1!}\left[\frac{\partial q(x,t)}{\partial t}\right]_{(0,0)}=-17*22.6\left(\sec h^2[x+22.6t]\right)\tanh[x+22.6t]=0 \tag{79}$$

$$
\begin{aligned}
Q(2,0)&=\frac{1}{2!0!}\left[\frac{\partial^2 q(x,t)}{\partial x^2}\right]_{(0,0)}\\
&=\frac{1}{2!}\left[-17\left(\sec h^4[x+22.6t]\right)-17\left(\sec h^2[x+22.6t]\right)\tanh^2[x+22.6t]\right]=-8.5
\end{aligned} \tag{80}
$$

$$
\begin{aligned}
Q(0,2)=\frac{1}{0!2!}\left[\frac{\partial^2 q(x,t)}{\partial t^2}\right]_{(0,0)}&=\frac{1}{2!}\left[-384.2*22.6\left(\sec h^4[x+22.6t]\right)\right.\\
&\left.+384.2*22.6\left(\sec h^2[x+22.6t]\right)\tanh^2[x+22.6t]\right]=-4341.46
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
Q(1,1)&=\frac{1}{1!1!}\left[\frac{\partial^2 q(x,t)}{\partial x.\partial t}\right]_{(0,0)}\\
&=\frac{1}{2!}\left[-17*22.6\left(\sec h^4[x+22.6t]\right)+17*2*22.6\left(\sec h^2[x+22.6t]\right)\tanh^2[x+22.6t]\right]\\
&=-384.2
\end{aligned} \tag{82}
$$

Using the initial conditions and the recurrence relation, taking $k=0\&h=0$ in Eq. (75) to get $Q(4,0)$

$$Q(4,0)=\frac{-11.4Q(1,1)+242.94*2Q(2,0)+32.82Q(0,0)-F(0,0)}{24}$$

where $F(0,0)$ given from Eq. (74) as

**Table 1** The values of the coefficients Q(k, h) for real part with k = 0,..,4 and h = 0,…,4

| k | h | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 0 | 8.5 | 0 | −4341.46 | 0 | $1.478296 * 10^6$ |
| 1 | 0 | −384.2 | 0 | 261645.3226 | 0 |
| 2 | −8.5 | 0 | 17365.84 | 0 | $-1.25655 * 10^7$ |
| 3 | 0 | 513 | 0 | −738423 | 0 |
| 4 | 5.554 | 0 | −24655.8 | 0 | $4.74545 * 10^7$ |

$$F(0,0) = (Q(0,0))^3 = (8.5)^3 = 614.125$$

Using the initials $Q(1,1) = -384.2 \& Q(2,0) = -8.5$ and the previous equation then

$$Q(4,0) = 5.554$$

In the recurrence relation putting $k = 0, h = 1$ in Eq. (75) to get $Q(4,1)$

$$Q(4,1) = \frac{-11.4Q(1,2) + 242.94 * 2Q(2,1) + 32.82Q(0,1) - F(0,1)}{24}$$

where $F(0,1) = 3Q(0,1)(Q(0,0))^2 = 0, Q(1,2) = Q(2,1) = Q(0,1) = 0$ then

$$Q(4,1) = 0$$

By the same we can calculate the remainder of the required coefficients which can be inscribed in the following table with $k = 0,1,2,3,4 andh = 0,1,2,3,4$ as (Table 1).

Then substituting in the solution formula in Eq. (65) we have

$$\begin{aligned} q(x,t) &= \sum_{k=0}^{4}\sum_{h=0}^{4} Q(k,h)x^k t^h \\ &= 8.5 - 4341.46t^2 + 1.478296 \times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x \\ &\quad + \left(-8.5 + 17365t^2 - 1.2565516 \times 10^7 t^4\right)x^2 \\ &\quad + \left(513t - 738423t^3\right)x^3 + \left(5.554 - 24655.8t^2 + 4.74545 \times 10^7 t^4\right)x^4 \end{aligned}$$

And the TDDTM for the real part of the Schrödinger–Poisson dynamical system is

$$\begin{aligned} \mathrm{Re}\Psi(x,t) &= q(x,t) \times \cos(5.7x - 14.5t) \\ &= \left(8.5 - 4341.46t^2 + 1.478296 \times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x \right. \\ &\quad + \left(-8.5 + 17365t^2 - 1.2565516 \times 10^7 t^4\right)x^2 + \left(513t - 738423t^3\right)x^3 \\ &\quad \left. + \left(5.554 - 24655.8t^2 + 4.74545 \times 10^7 t^4\right)x^4\right) \times \cos(5.7x - 14.5t) \end{aligned} \tag{83}$$

The Taylor series expansion of the exact solution is

$$\begin{aligned} q(x,t) &= 8.5\left(sech^2[x + 22.6t]\right) \\ &= 8.5 - 4341.46t^2 + 1.478296 \times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x \\ &\quad + \left(-8.5 + 17365t^2 - 1.2565516 \times 10^7 t^4\right)x^2 + \left(513t - 738423t^3\right)x^3 \\ &\quad + \left(5.554 - 24655.8t^2 + 4.74545 \times 10^7 t^4\right)x^4 \end{aligned}$$

Then The Taylor series expansion of the exact bright wave solution to the real part the Schrödinger–Poisson dynamical system is

$$\begin{aligned}\mathrm{Re}\Psi(x,t) &= q(x,t)\times\cos(5.7x-14.5t) = 8.5\left(\sec h^2[x+22.6t]\right)\times\cos(5.7x-14.5t)\\ &= \left(8.5-4341.46t^2+1.478296\times10^6t^4+\left(-384.2t+261645.3226t^3\right)x\right.\\ &\quad+\left(-8.5+17365t^2-1.2565516\times10^7t^4\right)x^2+\left(513t-738423t^3\right)x^3\\ &\quad\left.+\left(5.554-24655.8t^2+4.74545\times10^7t^4\right)x^4\right)\times\cos(5.7x-14.5t)\end{aligned} \tag{84}$$

From Eqs. (83) and (84) we note the similarity and equality of the TDDTM in Eq. (80) and the closed form using Taylor series of the exact bright wave solution of the Schrödinger–Poisson dynamical system in Eq. (81) which proves the efficiency and simplicity of the TDDTM.

### 4.1 TDDTM solution for the imaginary part of the Schrödinger–Poisson dynamical system

Consider the imaginary part of the Schrödinger–Poisson dynamical system Eq. (12)

$$4kq_{xxx}-q_{xxt}+k^2q_t-(2\Omega k+k^3+2\Phi k)q_x+2\Phi' qk=0$$

Taking the same values of the parameters given before $k=5.7,\Omega=14.5,\Phi=33.5,\Phi'=0$ then the previous equation becomes

$$22.8q_{xxx}-q_{xxt}+32.94q_t-732.393q_x=0(86) \tag{85}$$

Using the relations given in rule (1), we have

$$\begin{aligned}&q_{xxx}\rightarrow(k+1)(k+2)(k+3)Q(k+3,h)\\ &q_{xxt}\rightarrow(k+1)(k+2)(h+1)Q(k+2,h+1)\\ &q_x\rightarrow(k+1)Q(k+1,h)\&q_t(x,t)\rightarrow(h+1)Q(k,h+1)\end{aligned} \tag{86}$$

Substituting from Eq. (83) in Eq. (82) we have

$$\begin{aligned}&22.8(k+1)(k+2)(k+3)Q(k+3,h)\\ &-(k+1)(k+2)(h+1)Q(k+2,h+1)\\ &+32.94(h+1)Q(k,h+1)\\ &-732.393(k+1)Q(k+1,h)=0\end{aligned}$$

And the recurrence relation takes the form

$$Q(k+3,h) = \frac{(k+1)(k+2)(h+1)Q(k+2,h+1)-32.94(h+1)Q(k,h+1)+732.393(k+1)Q(k+1,h)}{22.8(k+1)(k+2)(k+3)} \tag{87}$$

Taking the exact first solution reduced from the bright solitary wave,

$$q(x,t)=8.5\left(sech^2[x+22.6t]\right)$$

Since the solution is the same as in the real case, so using the initial conditions and recurrence relation Eq. (87) we get on the TDDTM in the form;

$$
\begin{aligned}
q(x,t) &= \sum_{k=0}^{4}\sum_{h=0}^{4} Q(k,h)x^k t^h \\
&= 8.5 - 4341.46t^2 + 1.478296\times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x \\
&\quad + \left(-8.5 + 17365t^2 - 1.2565516\times 10^7 t^4\right)x^2 + \left(513t - 738423t^3\right)x^3 \\
&\quad + \left(5.554 - 24655.8t^2 + 4.74545\times 10^7 t^4\right)x^4
\end{aligned}
$$

And the TDDTM for the real part of the Schrödinger–Poisson dynamical system is

$$
\begin{aligned}
Im\Psi(x,t) &= q(x,t)\times sin(5.7x - 14.5t) \\
&= (8.5 - 4341.46t^2 + 1.478296\times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x \\
&\quad + \left(-8.5 + 17365t^2 - 1.2565516\times 10^7 t^4\right)x^2 + \left(513t - 738423t^3\right)x^3 \\
&\quad + \left(5.554 - 24655.8t^2 + 4.74545\times 10^7 t^4\right)x^4) \times sin(5.7x - 14.5t)
\end{aligned} \tag{88}
$$

The Taylor series expansion of the exact bright wave solution is

$$
\begin{aligned}
Im\Psi(x,t) &= q(x,t) = 8.5\left(sech^2[x + 22.6t]\right) \\
&= 8.5 - 4341.46t^2 + 1.478296\times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x \\
&\quad + \left(-8.5 + 17365t^2 - 1.2565516\times 10^7 t^4\right)x^2 + \left(513t - 738423t^3\right)x^3 \\
&\quad + \left(5.554 - 24655.8t^2 + 4.74545\times 10^7 t^4\right)x^4
\end{aligned}
$$

Then the Taylor series expansion of the exact bright wave solution to the real part of the Schrödinger–Poisson dynamical system is

$$
\begin{aligned}
Im\Psi(x,t) &= q(x,t)\times \sin(5.7x - 14.5t) \\
&= 8.5\left(\sec h^2[x + 22.6t]\right)\times \sin(5.7x - 14.5t) \\
&= \left(8.5 - 4341.46t^2 + 1.478296\times 10^6 t^4 + \left(-384.2t + 261645.3226t^3\right)x\right. \\
&\quad + \left(-8.5 + 17365t^2 - 1.2565516\times 10^7 t^4\right)x^2 + \left(513t - 738423t^3\right)x^3 \\
&\quad \left. + \left(5.554 - 24655.8t^2 + 4.74545\times 10^7 t^4\right)x^4\right) \times \sin(5.7x - 14.5t)
\end{aligned} \tag{89}
$$

From Eq. (88) and Eq. (89) we note the similarity and equality of the TDDTM in Eq. (88) and the closed form using Taylor series of the exact solution Eq. (89) which proves the efficiency and simplicity of the TDDTM.

By the same technique we can implement the numerical solutions of the other achieved dark solutions.

## 5 Conclusion

The Schrödinger–Poisson dynamical system which applied in gravity field with the corresponding quantum state that produce coupling between different gravity states is one of very effective models in physics and applied mathematics. In fact, it responsible the charges distribution on material that signify if it conductor or insulated. Hereby, the SWAM has been used for the first time to extract the bright and dark optical soliton solutions which weren't realized before. The suggested method admits new forms of the achieved soliton solutions as M-shaped soliton and W-shaped as well as bright and dark soliton solutions. The amplitude of the bright solutions is real while amplitude of the dark solutions is complex but this contradiction is acceptable because the intensity of the wave is directly proportion with the square of the amplitude whatever the values of choose parameters. The obtained solutions show the regular charge distribution for all conducting materials as well as its show significant the quantum state that produces coupling between different gravity states. The novelties of our obtained solutions appear when we compared it's with that realized before by Younis et al. (2021) who use (G'/G)-expansion method and the extended direct algebraic methods. Furthermore, the numerical solutions corresponding to the achieved analytical solutions have been constructed by using the two-dimensional differential transform method. The compatible between the analytical and the numerical solutions clears because there exists agreement between the McLaurin expansion to the achieved exact solutions, the obtained numerical solutions and we demonstrated this in the above table. These new types of solutions will play many fundamental roles in the quite understanding of qualitative features of many phenomenon arising in various areas of natural science and will add future studies for all related phenomenon in plasma physics, applied mathematics and engineering.

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## Declarations

**Conflict of interest** The authors declare that they have no competing interest.

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## References

Ayaz, F.: On the two-dimensional differential transform method. Appl. Math. Comput. **143**, 361–374 (2003)

Bekir, A., Zahran, E.H.M.: Bright and dark soliton solutions for the complex Kundu-Eckhaus equation. Optik **223**, 165233 (2020)

Bekir, A., Zahran, E.H.M.: New multiple-different impressive perceptions for the solitary solution to the magneto-optic waveguides with anti-cubic nonlinearity. Optik **240**, 166939 (2021)

Bekir, A., Zahran, E.H.M.: Three distinct and impressive visions for the soliton solutions to the higher-order nonlinear Schrödinger equation. Optik **228**, 166157 (2021a)

Bekir, A., Zahran, E.M.H.: Exact and numerical solutions for the Nano-Soliton of Ionic Wave propagating through microtubules in living cells. Pramana J. Phys. **95**, 158 (2021b)

Bekir, A., Zahran, E.H.M.: New multiple-different impressive perceptions for the solitary solution to the magneto-optic waveguides with anti-cubic nonlinearity. Optik **240**, 166939 (2021c)

Bekir, A., Zahran, E.H.M.: New vision for the soliton solutions to the complex Hirota-dynamical model. Phys. Scr. **96**, 055212 (2021d)

Bekir, A., Zahran, E.H.M.: New visions of the soliton solutions to the modified nonlinear Schrodinger equation. Optik **232**, 166539 (2021e)

Bekir, A., Zahran, E.H.M.: New multiple-different impressive perceptions for the solitary solution to the magneto-optic waveguides with anti-cubic nonlinearity. Optik – Int. J. Light Electron Opt. **240**, 166939 (2021f)

Bekir, A., Zahran, E.H.M.: Three distinct and impressive visions for the soliton solutions to the higher-order nonlinear Schrödinger equation. Optik – Int. J. Light Electron Opt. **228**, 166157 (2021g)

Bekir, A., Shehata, M.S.M., Zahran, E.M.H.: New optical soliton solutions for the thin-film ferroelectric materials equation instead of the numerical solution. Comput. Methods Differ. Equ. **9**(1), 158–167 (2020)

Bekir, A., Zahran, E.M.H., Shehata, M.S.M.: Comparison between the new exact and numerical solutions of the Mikhailov-Novikov-Wang equation. Numer. Method Part. Differ. Equ. J. (2021). https://doi.org/10.1002/num.22775

Chen, C., Ho, S.: Solving Partial Differential Equations by two-dimensional differential transform method. Appl. Math. Comput. **106**, 171–179 (1999)

Chen, S.S., Tian, B., Qu, Q.X., Li, H., Sun, Y., Du, X.X.: Alfvén solitons and generalized Darboux transformation for a variable-coefficient derivative nonlinear Schrödinger equation in an inhomogeneous plasma. Chaos Solitons Fract. **148**, 111029 (2021)

Giulini, D., Großardt, A.: The Schrödinger-Newton equation as a non-relativistic limit of self-gravitating Klein-Gordon and dirac fields. Class Quant. Grav. **29**(21), 215010 (2012)

Hosseini, K., Osman, M.S., Mirzazadeh, M., Rabiei, F.: Investigation of different wave structures to the generalized third-order nonlinear Schrödinger equation. Optik **206**, 164259 (2020)

Liu, J.G., Zhu, W.H., Zhou, L., Xiong, Y.K.: Multi-waves, breather wave and lump-stripe interaction solutions in a (2+1)-dimensional variable-coefficient Korteweg-de Vries equation. Nonlinear Dyn. **97**, 2127–2134 (2019)

Mao, J.J., Tian, S.F., Zou, L., Zhang, T.T., Yan, X.J.: Bilinear formalism, lump solution, lumpoff and instanton/rogue wave solution of a (3+1)-dimensional B-type Kadomtsev-Petviashvili equation. Nonlinear Dyn. **95**, 3005–3017 (2019)

Mirzazadeh, M., Yıldırım, Y., Yaşar, E., Triki, H., Zhou, Q., Moshokoa, S.P., Zaka Ullah, M., Seadawy, A.R., Biswas, A., Belic, M.: Optical solitons and conservation law of Kundu-Eckhaus equation. Optik **154**, 551–557 (2018)

Nakatsuji, H.: Discovery of a general method of solving the Schrodinger and dirac equations that opens a way to accurately predictive quantum chemistry. Acc. Chem. Res. **45**(9), 1480–1490 (2012)

Patel, Y.F., Dhodiya, J.M.: Application of differential transform method to real world problems, 1st edn. Chapman and Hall/CRC, Boca Raton (2022)

Penrose, R.: On gravity's role in quantum state reduction. Gen. Relat. Grav. **28**(5), 581–600 (1996)

Rehman, H.U., Younis, M., Jafar, S., Tahir, M., Saleem, M.S.: Optical solitons of Biswas-Arshed model in birefrigent fiber without four wave mixing. Optik **213**, 164669 (2020)

Schrödinger, E.: Quantization as an eigenvalue problem. Ann. Phys. **384**(4), 273–376 (1926)

Shehata, M.S.M., Rezazadeh, H., Zahran, E.H.M., Tala-Tebue, E., Bekir, A.: New Optical soliton solutions of the perturbed Fokas-Lenells Equation. Commun. Theor. Phys. **71**, 1275–1280 (2019)

Wang, X., Wei, J.: Antidark solitons and soliton molecules in a (3+1)-dimensional nonlinear evolution equation. Nonlinear Dyn. **102**, 363–377 (2020)

Younis, M., Sulaiman, T.A., Bilal, M., Rehman, S.U., Younas, U.: Modulation instability analysis optical and other solutions to the modified nonlinear Schrödinger equation. Commun. Theor. Phys. **72**, 065001 (2020)

Younis, M., Seadawy, A.R., Baber, M.Z., Husain, S., Iqbal, M.S., Rizvi, S.T.R., Baleanu, D.: Analytical optical soliton solutions of the Schrödinger–Poisson dynamical system. Results Phys. **27**, 104369 (2021)

Zahran, E.H.M., Bekir, A.: New private types for the cubic-quartic optical solitons in birefringent fibers in its four forms of nonlinear refractive index. Opt. Quant. Electron. **53**, 680 (2021)

Zahran, E.H.M., Bekir, A.: Multiple accurate-cubic optical solitons to the kerr-law and power-law nonlinear Schrödinger equation without the chromatic dispersion. Opt. Quant. Electron. **54**, 14 (2022)

Zahran, E.H.M., Bekir, A., Alotaibi, M.F., Omri, M., Ahmed, H.: New impressive behavior of the exact solutions to the Benjamin-Bona-Mahony-Burgers equation with dual power-law nonlinearity against its numerical solution. Results Phys. **29**, 104730 (2021a)

Zahran, E.H.M., Bekir, A., Ahmed, H.: A variety of exact solutions of the (2+1)-dimensional modified Zakharov-Kuznetsov equation. Modern Phys. Lett. B **35**, 2150509 (2021)

Zhou, J.: Differential transformation and its applications for electrical circuits. Borneo Huazhong University Press, Wuhan, China (2010)

Ziyaee, F., Tari, A.: Differential transform method for solving two-dimensional Fredholm integral equation. Appl. Appl. Math. J. **10**(2), 14 (2015)